Int J Theor Phys (2007) 46: 2901–2909
DOI 10.1007/s10773-007-9401-7

Entangled States in the Capacitance Coupling Double Josephson Junction Mesoscopic Circuit

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Published online: 15 May 2007

Abstract The number-phase quantization scheme of the capacitance coupling double Josephson junction mesoscopic circuit is given. The eigenvalues and the eigenstates of the system are investigated. It is found that using this system the entangled states can be prepared.

Keywords Josephson junction · Capacitance coupling · Quantization · Entangled state

1 Introduction

In recent years, with the progress of the theory of quantum information and quantum computation, the physical research on mesoscopic systems caused great interest [14, 15]. A practical quantum computer, if built, would be comprised of a set of coupled two-state quantum systems [10], usually referred as quantum bits (qubits), whose coherent time evolution must be controlled in a computation [6]. Experimentally, nuclear magnetic resonance in molecules [13], trapped ions [4], and quantum optical systems [12] have been investigated for embodying quantum computation. These systems have the advantages of high quantum coherence, but cannot be integrated easily to form large-scale circuits.

The macroscopic quantum effects in low-capacitance Josephson junction circuits have been used to realize qubits for quantum information processing [8], and these qubits are promising for quantum circuit integration for quantum computation. The charge qubit is achieved in a Cooper-pair island, where, under some certain conditions, only two charge states play an important role. It has been recognized using these circuits the logic operations could be performed by controlling gate voltages or external magnetic fields [5, 6, 8]. Theoretically and experimentally, the simplest model of Josephson junction charge qubit was

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proposed earlier [1, 11]. Subsequently, Makhlin et al. put forward the improved ideal design [6]. This design avoided an undesired phase evolution effectively, by tuning the energy splitting between logical states during idle periods between operations. The next challenge involved is to realize two-qubit gates by coupling charge qubits in a feasible way, since the single qubit and two qubits controlled NOT (CNOT) gates together can be used to implement an arbitrary two-level unitary operation on the state space of n qubits [9]. In Ref. [16], the writers proposed a scheme to couple charge qubits by mutual inductance and obtained some instructive conclusions. Furthermore, for any system, the existence of entangled states is at the heart of quantum information processing. Thus, it is significant to investigate how to couple charge qubit structures and whether the entangled states can be prepared in the coupling system.

In this paper, the entangled states of the capacitance coupling double Josephson junction mesoscopic circuit can be prepared. Firstly, we give the quantization scheme of the capacitance coupling double Josephson junction mesoscopic circuit. Secondly, we exactly formulate the Hamiltonian operator of the system in compact forms in spin-1/2 notation. Finally, we investigate the entangled states which exist in this system.

2 Quantization of the Capacitance Coupling Double Josephson Junction Mesoscopic Circuit

For the mesoscopic circuit with two Josephson junctions (see Fig. 1), it consists of three different elementary sections: two single-qubit structures and a coupling capacitance C_c. For each single-qubit structure, a superconducting island is coupled by a Josephson junction (each with capacitance C_j and Josephson coupling energy E_j) to another superconductor and through a capacitance C_{gi} to a voltage source V_{gi}.

In Ref. [3], Feynman et al. gave

$$\dot{\varphi}_{ji} = \frac{2eu_{ji}}{\hbar} \quad (i = 1, 2), \tag{1}$$

where u_{ji} is the electric potential difference across the Josephson junction. So when tunneling from one side to another, the variation of Cooper-pair's potential energy is

$$-\int_0^t u_{ji} I_c \sin\varphi_i dt = E_j(1 - \cos\varphi_i) \quad (i = 1, 2), \tag{2}$$

where $I_c = 2eE_j/\hbar$ is the critical electric current of the Josephson junction. Thus, the potential energy related to the generalized coordinates φ_1 and φ_2 of the system is

$$V = \sum_{i=1,2} E_j(1 - \cos\varphi_i). \tag{3}$$

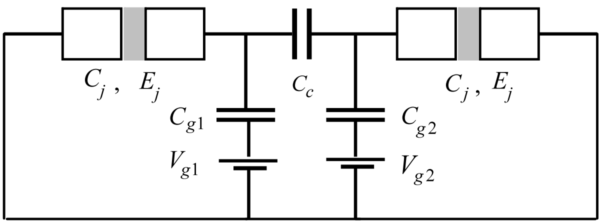

Fig. 1 The mesoscopic circuit with two Josephson junctions

The kinetic energy related to $\dot{\varphi}_i$ is

$$T = \sum_{i=1,2} \left[\frac{1}{2} C_{\Sigma i} \left(\frac{\hbar}{2e} \right)^2 \dot{\varphi}_i^2 + \hbar N_{gi} \dot{\varphi}_i + \frac{2N_{gi}^2 e^2}{C_{gi}} \right] - C_c \left(\frac{\hbar}{2e} \right)^2 \dot{\varphi}_1 \dot{\varphi}_2, \tag{4}$$

where $N_{gi} = -\frac{C_{gi} V_{gi}}{2e}$, $C_{\Sigma i} = C_j + C_c + C_{gi}$. Then the Lagrangian function of the system can be obtained

$$\ell = T - V = \sum_{i=1,2} \left[\frac{1}{2} C_{\Sigma i} \left(\frac{\hbar}{2e} \right)^2 \dot{\varphi}_i^2 + \hbar N_{gi} \dot{\varphi}_i + \frac{2N_{gi}^2 e^2}{C_{gi}} - E_j (1 - \cos \varphi_i) \right] - C_c \left(\frac{\hbar}{2e} \right)^2 \dot{\varphi}_1 \dot{\varphi}_2. \tag{5}$$

Through the Josephson junction, Cooper-pairs can tunnel from or onto the island. The number n_i $(i = 1, 2)$ of the excess Cooper-pairs on the island depends on the gate voltage V_{gi}. The number n_i satisfies the following relational expression

$$2n_1 e = \frac{\hbar}{2e} C_{\Sigma 1} \dot{\varphi}_1 - \frac{\hbar}{2e} C_c \dot{\varphi}_2 + 2N_{g1} e, \qquad 2n_2 e = \frac{\hbar}{2e} C_{\Sigma 2} \dot{\varphi}_2 - \frac{\hbar}{2e} C_c \dot{\varphi}_1 + 2N_{g2} e. \tag{6}$$

The generalized momenta conjugated to φ_1 and φ_2 are, respectively,

$$p_1 = \frac{\partial \ell}{\partial \dot{\varphi}_1} = \frac{\hbar}{2e} \left[\frac{\hbar}{2e} C_{\Sigma 1} \dot{\varphi}_1 - \frac{\hbar}{2e} C_c \dot{\varphi}_2 + 2N_{g1} e \right] = n_1 \hbar, \tag{7a}$$

$$p_2 = \frac{\partial \ell}{\partial \dot{\varphi}_2} = \frac{\hbar}{2e} \left[\frac{\hbar}{2e} C_{\Sigma 2} \dot{\varphi}_2 - \frac{\hbar}{2e} C_c \dot{\varphi}_1 + 2N_{g2} e \right] = n_2 \hbar. \tag{7b}$$

Note in particular that the generalized momentum p_i is proportional to n_i.

Thus, from (5), (6) and (7), we can obtain the classical Hamiltonian of the system

$$H = H_1 + H_2 + H_{\text{int}}, \tag{8}$$

where

$$H_i = 4E_{c,j}^{(i)} (n_i - N_{gi})^2 + E_j (1 - \cos \varphi_i) - 2N_{gi}^2 e^2 / C_{gi} \quad (i = 1, 2), \tag{9a}$$

$$H_{\text{int}} = 4E_{c,c} \prod_{i=1,2} (n_i - N_{gi}), \tag{9b}$$

$$E_{c,j}^{(i)} = \frac{e^2}{2C_{j,\text{eff}}^{(i)}}, \qquad E_{c,c} = \frac{e^2}{2C_{c,\text{eff}}}, \tag{10}$$

$$C_{j,\text{eff}}^{(1)} = C_{\Sigma 1} - \frac{C_c^2}{C_{\Sigma 2}}, \qquad C_{j,\text{eff}}^{(2)} = C_{\Sigma 2} - \frac{C_c^2}{C_{\Sigma 1}}, \qquad C_{c,\text{eff}} = \frac{C_{\Sigma 1} C_{\Sigma 2}}{C_C} - C_C, \tag{11}$$

where H_i $(i = 1, 2)$ is the Hamiltonian of the single qubit structure, H_{int} is the coupling term and $E_{c,j}^{(i)} = e^2 / (2C_{j,\text{eff}}^{(i)})$ is the single electron charging energy on the island. According to the standard quantization principle, a pair of conjugated quantities n_i and φ_i is associated with a pair of Hermitian operators $\hat{n}_i$ and $\hat{\varphi}_i$, respectively. They satisfy the commutation relation $[\hat{n}_i, \hat{\varphi}_i] = -\mathrm{i}$. Thus, the quantized Hamiltonian of the system can be written as

$$\hat{H} = \hat{H}_1 + \hat{H}_2 + \hat{H}_{\text{int}}, \tag{12}$$

where

$$\hat{H}_i = 4E_{c,j}^{(i)}(\hat{n}_i - N_{gi})^2 + E_j(1-\cos\hat{\varphi}_i) - 2N_{gi}^2 e^2/C_{gi}, \quad (i=1,2), \tag{13a}$$

$$\hat{H}_{\text{int}} = 4E_{c,c}\prod_{i=1,2}(\hat{n}_i - N_{gi}). \tag{13b}$$

3 Hamiltonian Operator of the System in Spin-1/2 Notation

Now we discuss the various contributions to $\hat{H}$ given by (12) in detail. Here, we choose the material such that the superconducting energy gap Δ is the largest energy in the problem, larger even than the single electron charging energy $E_{c,j}^{(i)}$ on the island. In this case quasiparticle tunneling is suppressed at low temperature, and a situation can be reached in which no quasiparticle excitation is found on the island [7]. Moreover, we consider the system in which the single electron charging energy is much larger than the Josephson coupling energy, i.e., $E_{c,j}^{(i)} \gg E_j$. In this case, a convenient basis is formed by charge states, parameterized by the excess number n_i of Cooper-pairs on the island [7]: $\hat{n}_i|n\rangle_i = n_i|n\rangle_i$.

Charge Qubit-1 Structure From the commutation relation $[\hat{n}_1, \hat{\varphi}_1] = -\mathrm{i}$, we obtain

$$[\hat{n}_1, \exp(\mathrm{i}\hat{\varphi}_1)] = \exp(\mathrm{i}\hat{\varphi}_1). \tag{14}$$

Using (14) and the basis state $|n\rangle_1$, we have

$$\exp(\mathrm{i}\hat{\varphi}_1)|n\rangle_1 = |n+1\rangle_1. \tag{15a}$$

Similarly, we have

$$\exp(-\mathrm{i}\hat{\varphi}_1)|n\rangle_1 = |n-1\rangle_1. \tag{15b}$$

Thus, we can deduce

$$\cos\hat{\varphi}_1 = \frac{1}{2}\sum_n(|n+1\rangle_{11}\langle n| + |n-1\rangle_{11}\langle n|). \tag{16}$$

In the condition of the low temperature, i.e., $k_B T \ll E_{c,j}^{(i)}$ [1], we concentrate on such a voltage near a degeneracy point where only two charge states $|0\rangle$ and $|1\rangle$ play an important role. While all other charge states, having a much higher energy, can be ignored [7]. In this case, from (16) we obtain

$$\cos\hat{\varphi}_1 = \frac{1}{2}(|0\rangle_{11}\langle 1| + |1\rangle_{11}\langle 0|) = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix}. \tag{17}$$

Thus, the operator $\hat{H}_1$ can be presented as a matrix

$$\hat{H}_1 = \begin{pmatrix} k_1 & k_2 \\ k_2 & k_3 \end{pmatrix}, \tag{18}$$

where

$$k_1 = 4E_{c,j}^{(1)}N_{g1}^2 + E_j - 2N_{g1}^2 e^2/C_{g1}, \qquad k_2 = -E_j/2, \tag{19a}$$

$$k_3 = 4E_{c,j}^{(1)}(1-N_{g1})^2 + E_j - 2N_{g1}^2 e^2/C_{g1}. \tag{19b}$$

The eigenvalues of the operator $\hat{H}_1$ are

$$\lambda_1 = \frac{1}{2}\Big[(k_1 + k_3) + \sqrt{(k_1 - k_3)^2 + 4k_2^2}\,\Big], \qquad \lambda_1' = \frac{1}{2}\Big[(k_1 + k_3) - \sqrt{(k_1 - k_3)^2 + 4k_2^2}\,\Big], \tag{20}$$

and the corresponding eigenstates are, respectively,

$$|-\rangle_1 = \alpha_1|0\rangle_1 - \beta_1|1\rangle_1, \qquad |+\rangle_1 = \alpha_1'|0\rangle_1 + \beta_1'|1\rangle_1, \tag{21}$$

where $\alpha_1^2 = 1 - \beta_1^2 = k_2^2/[(\lambda_1 - k_1)^2 + k_2^2]$, $\alpha_1'^2 = 1 - \beta_1'^2 = (\lambda_1' - k_3)^2/[(\lambda_1' - k_3)^2 + k_2^2]$. The operator $\hat{H}_1$ can be presented in spin-1/2 notation as

$$\hat{H}_1 = \Gamma_1\hat{\sigma}_x^{(1)} + \Lambda_1\hat{\sigma}_z^{(1)} + \Xi_1, \tag{22}$$

where

$$\begin{aligned} &\Gamma_1 = -\frac{E_j}{2}, \qquad \Lambda_1 = 2E_{c,j}^{(1)}(2N_{g1} - 1),\\ &\Xi_1 = 2E_{c,j}^{(1)} - 4E_{c,j}^{(1)}N_{g1} + 4E_{c,j}^{(1)}N_{g1}^2 + E_j - \frac{2N_{g1}^2e^2}{C_{g1}}. \end{aligned} \tag{23}$$

For the special case of $N_{g1} = 1/2$ which is near degeneracy point, by a suitable manipulation of the gate voltage V_{g1}, the eigenstates of the operator $\hat{H}_1$ take the simple forms

$$|-\rangle_1 = \frac{|0\rangle_1 - |1\rangle_1}{\sqrt{2}}, \qquad |+\rangle_1 = \frac{|0\rangle_1 + |1\rangle_1}{\sqrt{2}}, \tag{24}$$

which are known as computational basis states [9] usually used in quantum computation and quantum information.

Charge Qubit-2 Structure The matrix form of the operator $\hat{H}_2$ is

$$\hat{H}_2 = \begin{pmatrix} \eta_1 & \eta_2 \\ \eta_2 & \eta_3 \end{pmatrix}, \tag{25}$$

where

$$\eta_1 = 4E_{c,j}^{(2)}N_{g2}^2 + E_j - 2N_{g2}^2e^2/C_{g2}, \qquad \eta_2 = -E_j/2, \tag{26a}$$

$$\eta_3 = 4E_{c,j}^{(2)}(1 - N_{g2})^2 + E_j - 2N_{g2}^2e^2/C_{g2}. \tag{26b}$$

The eigenvalues of the operator $\hat{H}_2$ are, respectively,

$$\lambda_2 = \frac{1}{2}\Big[(\eta_1 + \eta_3) + \sqrt{(\eta_1 - \eta_3)^2 + 4\eta_2^2}\,\Big], \qquad \lambda_2' = \frac{1}{2}\Big[(\eta_1 + \eta_3) - \sqrt{(\eta_1 - \eta_3)^2 + 4\eta_2^2}\,\Big], \tag{27}$$

and the corresponding eigenstates are, respectively,

$$|-\rangle_2 = \alpha_2|0\rangle_2 - \beta_2|1\rangle_2, \qquad |+\rangle_2 = \alpha_2'|0\rangle_2 + \beta_2'|1\rangle_2, \tag{28}$$

where $\alpha_2^2 = 1 - \beta_2^2 = k_2^2/[(\lambda_2 - \eta_1)^2 + \eta_2^2]$, $\alpha_2'^2 = 1 - \beta_2'^2 = (\lambda_2' - \eta_3)^2/[(\lambda_2' - \eta_3)^2 + \eta_2^2]$.

The operator $\hat{H}_2$ can be presented in spin-1/2 notation as

$$\hat{H}_2 = \varGamma_2 \hat{\sigma}_x^{(2)} + \varLambda_2 \hat{\sigma}_z^{(2)} + \varXi_2, \tag{29}$$

where

$$\begin{aligned} \varGamma_2 &= -\frac{E_j}{2}, \qquad \varLambda_2 = 2E_{c,j}^{(2)}(2N_{g2} - 1), \\ \varXi_2 &= 2E_{c,j}^{(2)} - 4E_{c,j}^{(2)} N_{g2} + 4E_{c,j}^{(2)} N_{g2}^2 + E_j - \frac{2N_{g2}^2 e^2}{C_{g2}}. \end{aligned} \tag{30}$$

For the special case of $N_{g2} = 1/2$ which is near degeneracy point, by a suitable manipulation of the gate voltage V_{g2}, the eigenstates of the operator $\hat{H}_2$ take the simple forms

$$|-\rangle_2 = \frac{|0\rangle_2 - |1\rangle_2}{\sqrt{2}}, \qquad |+\rangle_2 = \frac{|0\rangle_2 + |1\rangle_2}{\sqrt{2}}. \tag{31}$$

Coupling Term The coupling term $\hat{H}_{\text{int}}$ can be presented in spin-1/2 notation as

$$\hat{H}_{\text{int}} = \varLambda_3^{(1)} \hat{\sigma}_z^{(1)} + \varLambda_3^{(2)} \hat{\sigma}_z^{(2)} + \varLambda_3 \hat{\sigma}_z^{(1)} \hat{\sigma}_z^{(2)} + \varXi_3, \tag{32}$$

where

$$\begin{aligned} &\varLambda_3^{(1)} = E_{c,c}(2N_{g2} - 1), \qquad \varLambda_3^{(2)} = E_{c,c}(2N_{g1} - 1), \qquad \varLambda_3 = E_{c,c}, \\ &\varXi_3 = E_{c,c}(2N_{g1} - 1)(2N_{g2} - 1). \end{aligned} \tag{33}$$

Substituting (22), (29) and (32) into (12), we can presented the Hamiltonian operator $\hat{H}$ of the system in spin-1/2 notation as

$$\hat{H} = \varGamma_1 \hat{\sigma}_x^{(1)} + \varGamma_2 \hat{\sigma}_x^{(2)} + (\varLambda_1 + \varLambda_3^{(1)}) \hat{\sigma}_z^{(1)} + (\varLambda_2 + \varLambda_3^{(2)}) \hat{\sigma}_z^{(2)} + \varLambda_3 \hat{\sigma}_z^{(1)} \hat{\sigma}_z^{(2)} + \varXi_1 + \varXi_2 + \varXi_3. \tag{34}$$

Using these compact forms, one can conveniently describe the evolution of the system in the quantum computing language. According to the following unitary transformation

$$\hat{U}(\tau) = \exp(-\mathrm{i}\hat{H}\tau/\hbar), \tag{35}$$

the two-qubit operations can be realized.

For the special case of $N_{gi} = 1/2$ $(i = 1, 2)$ which is near the degeneracy point, (34) becomes

$$\hat{H} = -E_j/2\hat{\sigma}_x^{(1)} - E_j/2\hat{\sigma}_x^{(2)} + E_{c,c} \hat{\sigma}_z^{(1)} \hat{\sigma}_z^{(2)} + \varXi_1 + \varXi_2. \tag{36}$$

From (36), one can see that the quantum two-level system can be regarded as a spin-1/2 "particle". Here, the effective "magnetic field" is $B_x^{(1)} = B_x^{(2)} = E_j$ and the exchange energy between the two "particles" is $E_{c,c}$. Note that the Bohr magneto μ_B is in the definition of the "magnetic field" $B_x^{(i)}$. Because of the capacitance coupling, the Coulomb interaction leads to an Ising type [2] coupling term $\propto \sigma_z^{(1)} \sigma_z^{(2)}$. Such a coupling allows easy realization of a controlled-NOT operation [7].

4 The Eigenstates of the System and the Entanglement

We are interested in the situation $N_{gi} = 1/2$ ($i = 1, 2$) which is near degeneracy point. In this case, the system can be characterized by the four basis states $|+\rangle_1|+\rangle_2$, $|+\rangle_1|-\rangle_2$, $|-\rangle_1|+\rangle_2$ and $|-\rangle_1|-\rangle_2$. Using the above four basis states, the Hamiltonian of the system is expressed as

$$\hat{H} = \begin{pmatrix} E_0 & 0 & 0 & E_{c,c} \\ 0 & E_1 & E_{c,c} & 0 \\ 0 & E_{c,c} & E_1 & 0 \\ E_{c,c} & 0 & 0 & E_2 \end{pmatrix}, \tag{37}$$

which is a Hermitian matrix describing the two-level systems. Here,

$$E_0 = E_j + \sum_{i=1,2}\left[E_{c,j}^{(i)} - \frac{e^2}{2C_{gi}}\right], \qquad E_1 = 2E_j + \sum_{i=1,2}\left[E_{c,j}^{(i)} - \frac{e^2}{2C_{gi}}\right],$$
$$E_2 = 3E_j + \sum_{i=1,2}\left[E_{c,j}^{(i)} - \frac{e^2}{2C_{gi}}\right].$$

The eigenvalues of the Hamiltonian operator $\hat{H}$ are, respectively,

$$\varepsilon_1 = E_1 - E_{c,c}, \qquad \varepsilon_2 = E_1 + E_{c,c}, \tag{38a}$$

$$\begin{aligned} \varepsilon_3 &= \frac{1}{2}\Big[E_0 + E_2 - \sqrt{(E_0 - E_2)^2 + 4E_{c,c}^2}\Big], \\ \varepsilon_4 &= \frac{1}{2}\Big[E_0 + E_2 + \sqrt{(E_0 - E_2)^2 + 4E_{c,c}^2}\Big], \end{aligned} \tag{38b}$$

and the corresponding normalized eigenstates are, respectively,

$$|\psi_1\rangle = [|-\rangle_1|+\rangle_2 - |+\rangle_1|-\rangle_2]/\sqrt{2}, \tag{39a}$$

$$|\psi_2\rangle = [|-\rangle_1|+\rangle_2 + |+\rangle_1|-\rangle_2]/\sqrt{2}, \tag{39b}$$

$$|\psi_3\rangle = [|-\rangle_1|-\rangle_2 - \gamma_3|+\rangle_1|+\rangle_2]/\sqrt{1 + \gamma_3^2}, \tag{39c}$$

$$|\psi_4\rangle = [|-\rangle_1|-\rangle_2 - \gamma_4|+\rangle_1|+\rangle_2]/\sqrt{1 + \gamma_4^2}, \tag{39d}$$

where

$$\begin{aligned} \gamma_3 &= \frac{-E_0 + E_2 + \sqrt{(E_0 - E_2)^2 + 4E_{c,c}^2}}{2E_{c,c}}, \\ \gamma_4 &= \frac{-E_0 + E_2 - \sqrt{(E_0 - E_2)^2 + 4E_{c,c}^2}}{2E_{c,c}}. \end{aligned} \tag{40}$$

From (39a–39d), one can find, because of the capacitance coupling between the charge qubit structures, the charge qubits are entangled with each other. Note in particular that, if initially the system has been prepared in the state $|\psi(t = 0)\rangle = |-\rangle_1|+\rangle_2$, when $t \to 0^+$, it would be entangled with $|+\rangle_1|-\rangle_2$ maximally. The precise forms of the entanglement are governed by the central 2×2 block of the Hamiltonian matrix given by (37). At $t > 0$, the time evolution of the state describing the system is determined by the Hamiltonian matrix given by (37). From (38–39) and by virtue of the spectral decomposition [9], one can easily obtain

$$|\psi(t)\rangle = \exp(-\mathrm{i}\hat{H}t/\hbar)|-\rangle_1|+\rangle_2 = \frac{1}{\sqrt{2}}[\mathrm{e}^{-\mathrm{i}(E_1-E_{c,c})t/\hbar}|\psi_1\rangle + \mathrm{e}^{-\mathrm{i}(E_1+E_{c,c})t/\hbar}|\psi_2\rangle]. \quad (41)$$

From (41), it is found that the state $|\psi(t)\rangle$ oscillates coherently between $|-\rangle_1|+\rangle_2$ and $|+\rangle_1|-\rangle_2$, which is regarded as quantum Rabi oscillations. An interesting quantity is the probability distribution $P_1(t)$ to find the charge qubit-1 in the state $|-\rangle_1$ after a certain time t. The probability distribution $P_1(t)$ is

$$P_1(t) = |{}_1\langle -|{}_2\langle +|\psi(t)\rangle|^2 = \frac{1}{2}[1+\cos(2E_{c,c}t/\hbar)]. \quad (42)$$

From (42), one can see that the frequency of Rabi oscillation as a function of time is $2E_{c,c}/\hbar$. Since the oscillations are characteristic for the entanglement realized in the system, their measurement would provide direct evidence of the presence of the entangled states $|\psi_1\rangle$ and $|\psi_2\rangle$.

5 Conclusions

The number-phase quantization scheme of the capacitance coupling double Josephson junction mesoscopic circuit is proposed. Then, we exactly formulate the Hamiltonian operator of the system in compact forms in spin-1/2 notation. Using these compact forms, one can conveniently describe the evolution of the state of the systems in the quantum computation language. Also, we investigate the eigenstates of the system. It is found that, because of the capacitance coupling, the two charge qubits are entangled. The time evolution of the state of the system shows oscillation. Since the oscillations are characteristic for the entanglement realized in the system. Their measurement would provide direct evidence of the presence of the entangled states. From the view of quantum computation and quantum information, the above conclusions will be helpful to the study of entangled states.

Acknowledgements This work was supported by the National Natural Science Foundation of China (No. 10574060) and the Natural Science Foundation of Shandong Province of China (No. Y2004A09).

References

1. Bouchiat, V., Vion, D., Joyez, P., Esteve, D., Devoret, M.H.: Quantum coherence of charge states in the single electron box. J. Superconduct. **12**, 789 (1999)
2. Burkard, G., Loss, D., David, P.D., John, A.S.: Physical optimization of quantum error correction circuits. Phys. Rev. B **60**, 11404 (1999)
3. Feynman, R.P., Leighton, R.B., Sands, M.: The Feynman Lectures on Physics, vol. III. Addison–Wesley, Reading (1965)
4. Gulde, S., Riebe, M., Lancaster, G.P.T., Becher, C., Eschner, J., Häffner, H., Schmidt-Kaler, F., Chuang, I.L., Blatt, R.: Implementation of the Deutsch–Jozsa algorithm on an ion-trap quantum computer. Nature **421**, 48 (2003)
5. Ioffe, L.B., Geshkenbein, V.B., Feigel'man, M.V., Fauchère, A.L., Blatter, G.: Environmentally decoupled *sds*-wave Josephson junctions for quantum computing. Nature **398**, 679 (1999)
6. Makhlin, Y., Schön, G., Shnirman, A.: Josephson-junction qubits with controlled couplings. Nature **398**, 305 (1999)
7. Makhlin, Y., Schön, G., Shnirman, A.: Quantum-state engineering with Josephson junction devices. Rev. Mod. Phys. **73**, 357 (2001)

8. Nakamura, Y., Pashkin, Yu. A., Tsai, J.S.: Coherent control of macroscopic quantum states in a single-Cooper-pair box. Nature **398**, 786 (1999)
9. Nielsen, M.A., Chuang, I.L.: Quantum Computation and Quantum Information. Cambridge University Press, Cambridge (2000)
10. Pashkin, Yu. A., Yamamoto, T., Astafiev, O., Nakamura, Y., Averin, D.V., Tsai, J.S.: Quantum oscillations in two coupled charge qubits. Nature **421**, 823 (2003)
11. Shirman, A., Schön, G., Hermon, Z.: Quantum manipulations of small Josephson junctions. Phys. Rev. Lett. **79**, 2371 (1997)
12. Turchette, Q.A., Hood, C.J., Lange, W., Mabuchi, H., Kimble, H.J.: Measurement of conditional phase shifts for quantum logic. Phys. Rev. Lett. **75**, 4710 (1995)
13. Vandersypen, L.M.K., Steffen, M., Breyta, G., Yannoni, C.S., Sherwood, M.H., Chuang, I.L.: Experimental realization of Shor's quantum factoring algorithm using nuclear magnetic resonance. Nature **414**, 883 (2001)
14. Wang, J.S., Liu, T.K., Zhan, M.S.: Quantum fluctuations in a mesoscopic inductance coupling circuit. Int. J. Theor. Phys. **39**, 2013 (2000)
15. Wang, J.S., Liu, T.K., Zhan, M.S.: Quantum effects of a nondissipative mesoscopic capacitance coupling circuit in a displaced squeezed Fock state. Int. J. Theor. Phys. **39**, 2595 (2000)
16. You, J.Q., Lam, C.H., Zheng, H.Z.: Superconducting charge qubits: the roles of self and mutual inductances. Phys. Rev. B **63**, 180501 (2001)